TOWARDS A TRANSDISCIPLINARY FRAMEWORK TO ANALYZE BIRTH AND DEATH PATTERNS. DISCUSSION

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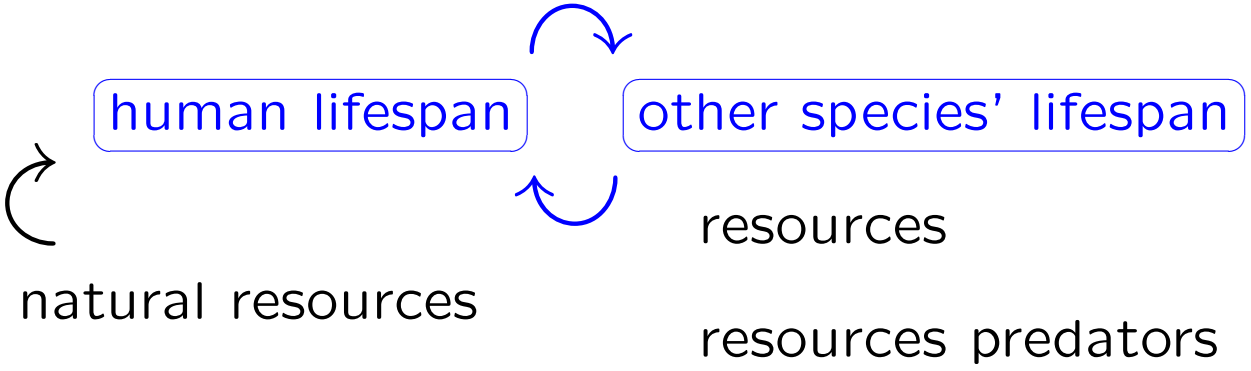
➠ *Understanding the nature and extent of* **biological** *constraints on the rate of ageing [..] is critical [...] to extend human lifespans...* (Colchero et al., 2021)

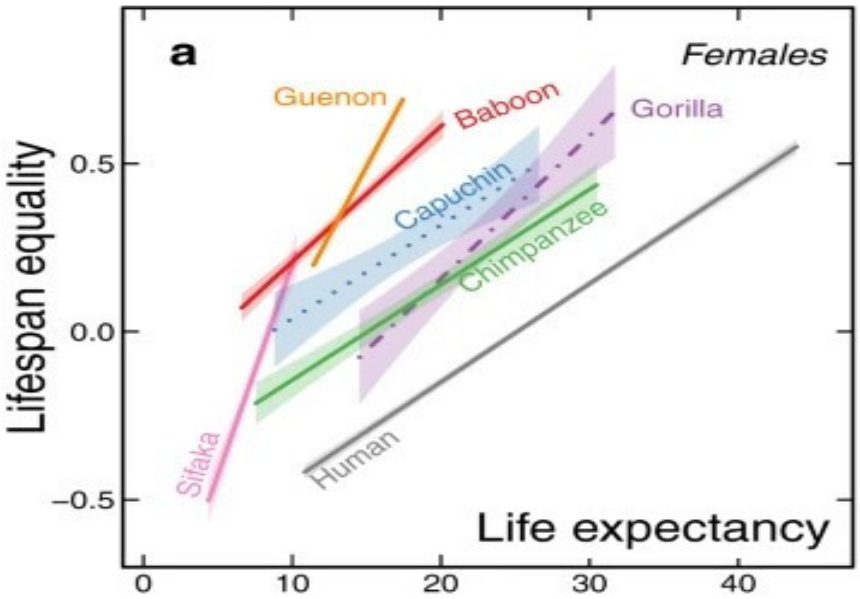
➠ *Under what conditions could the rate of ageing change?*

Colchero, F., et al. (2021). The long lives of primates and the 'invariant rate of ageing' hypothesis. Nature Communications, 12, 3666.

Pace & Shape as two different components of mortality

separate estimates ignore interactions?





DEVIATIONS MAY BE DUE TO EXTERNAL CONDITIONS

- interactions among species
  (e.g., human effect on environment)
- extemporaneous events
  (e.g., pandemic, war, extreme climatic events)
- limited resources

Colchero, F., et al. (2021). The long lives of primates and the 'invariant rate of ageing' hypothesis. Nature Communications, 12, 3666.

Alternative [unfeasible?] approach

3 species extended Lotka Volterra models; among others:

- Chauvet et al. (2002), Mathematics Magazine
- Hsu S.B. et. al.(2015) Journal of Mathematical Analysis and Applications

Chauvet, E., Paullet, J. E., Previte, J.P. and Walls, Z. (2002). A Lotka-Volterra Three-species Food Chain, *Mathematics Magazine*, 75(4), pp. 243-255.

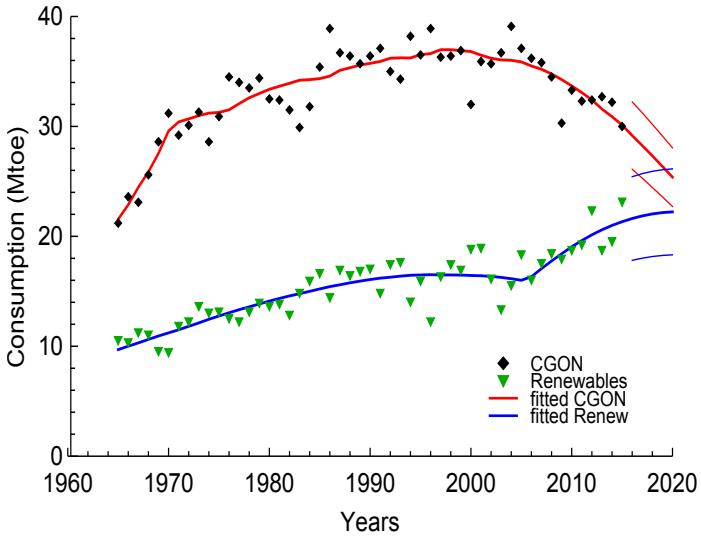
Hsu, S.B., Ruan S., Yan T.C. (2015). Analysis of three species Lotka–Volterra food web models with omnivory *Journal of Mathematical Analysis and Applications*, 426(2), pp. 659-687.

An example from a statistical point of view

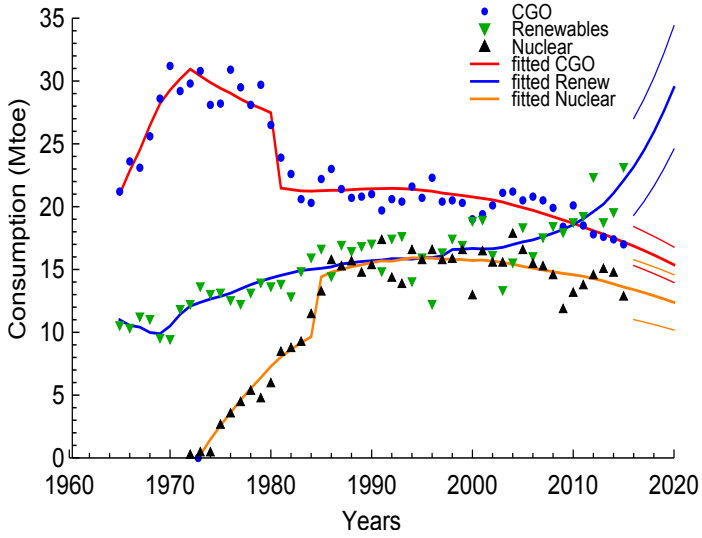
(Furlan et al., 2021)

Yearly energy consumptions (provided by BP, in Mtoe) for Sweden

CGON vs. Renewables



CGO vs N vs. Renewables



Furlan, C., Mortarino, C. and Zahangir, M. S. (2021). Interaction among three-substitute products: an extended innovation diffusion model. *Statistical Methods & Applications,* 30, pp. 269–293.

3CM (3 Competitor Model)

$$z_1'(t) = m\Big\{\Big[p_{1\alpha} + (q_{1\alpha} + \delta_\alpha)\frac{z_1(t)}{m} + q_{1\alpha}\frac{z_2(t)}{m}\Big](1 - I_{t>c_2}) +$$
$$+ \Big[p_{1\beta} + (q_{1\beta} + \delta_\beta)\frac{z_1(t)}{m} + q_{1\beta}\frac{z_2(t)}{m} + q_{1\beta}\frac{z_3(t)}{m}\Big] I_{t>c_2}\Big\} R(t)\ x_1(t)$$
$$z_2'(t) = m\Big\{\Big[p_{2\alpha} + (q_{2\alpha} - \delta_\alpha)\frac{z_1(t)}{m} + q_{2\alpha}\frac{z_2(t)}{m}\Big](1 - I_{t>c_2}) +$$
$$+ \Big[p_{2\beta} + q_{2\beta}\frac{z_1(t)}{m} + (q_{2\beta} + \delta_\beta)\frac{z_2(t)}{m} + q_{2\beta}\frac{z_3(t)}{m}\Big] I_{t>c_2}\Big\} R(t) x_2(t)$$
$$z_3'(t) = m\Big\{\Big[p_3 + (q_3 - \delta_\beta)\frac{z_1(t)}{m} + (q_3 - \delta_\beta)\frac{z_2(t)}{m} + q_3\frac{z_3(t)}{m}\Big] I_{t>c_2}\Big\} R(t) x_3(t)$$
$$m = m_\alpha(1 - I_{t>c_2}) + m_\beta I_{t>c_2}$$
$$z(t) = z_1(t) + z_2(t) + z_3(t) I_{t>c_2}$$