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Profile monitoring based on adaptive parameter learning

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Background

- Apply a sequential control statistic for **monitoring the stability** of incoming observations,

$$Y_t \sim \begin{cases} \mathbb{P}_{\mathbf{Y}}(\cdot, \theta_0) & \text{for } t = -m+1, \dots, 0, 1, \dots, \tau - 1 \\ \mathbb{P}_{\mathbf{Y}}(\cdot, \theta_1) & \text{for } t = \tau, \tau+1, \dots. \end{cases}$$

- If θ is the mean, an EWMA-type statistic can be applied for monitoring parameter increases,

$$C_t = \max\left\{0, (1-\lambda)C_{t-1} + \lambda(y_t - \mathbb{E}_{\theta_0}[Y_t])/(\sqrt{\mathbb{V}_{\theta_0}[Y_t]})\right\}$$

- Raise an alarm the first time that $C_t \geq L$, according to the following **stopping rule** $\mathrm{RL} = \inf\{t : C_t \geq L\}$.
- For a given critical value L, $\mathbb{E}[\mathrm{RL}|\tau = \infty] = \mathrm{ARL}_0$ and model performance is measured in terms of **ARL under shift**

$$\mathrm{ARL}_1 = \mathbb{E}[\mathrm{RL}|\tau = \tau_0].$$

- **Estimate** θ_0 using an initial sample of size m from the stable process.
- Set the limit L so that the **GICP property** [2] holds,

$$\mathbb{P}(\mathrm{ARL}_0 \leq a|\tau = \infty) = \beta, \qquad (1)$$

to account for model uncertainty.

Problem

Online monitoring phase: the process parameter at time t is either **held fixed** at $\widehat{\theta}_0$, or **adaptively estimated** with $\widehat{\theta}_{t-1}$ after checking that the process is stable.

- **Fixed-estimate** (FE): estimates have low precision, **but** can't be biased towards the unstable parameter value.
- **Adaptive-estimate** (AE): estimates have high precision, **but** could include unstable observations in the estimate, thereby reducing detection power.

Proposal

Enhancement of the Cautious Learning (CL) idea [1]

Combine the AE and FE algorithms in order to take advantage of their strengths and minimize their drawbacks.

- Study the properties of control statistics using a **time-delayed** parameter estimator $\widehat{\theta}_{t-\phi(t)}$.
- **Update the estimates** when the monitoring statistic gives **no evidence** of a parameter shift.
- **Stop the parameter updates** when there is evidence of a parameter shift.

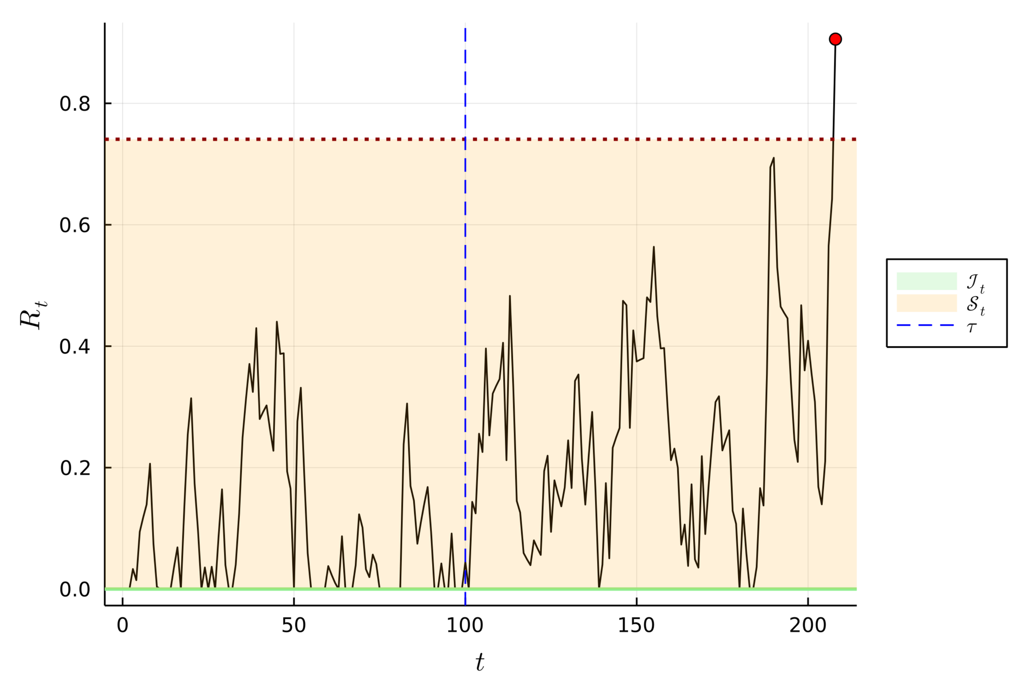

Figure 1. EWMA control chart: updating region (*green*) and warning region (*yellow*).

- Advantage: **lengthening the "window of opportunity"** to detect a parameter shift before it is masked by unstable observations.

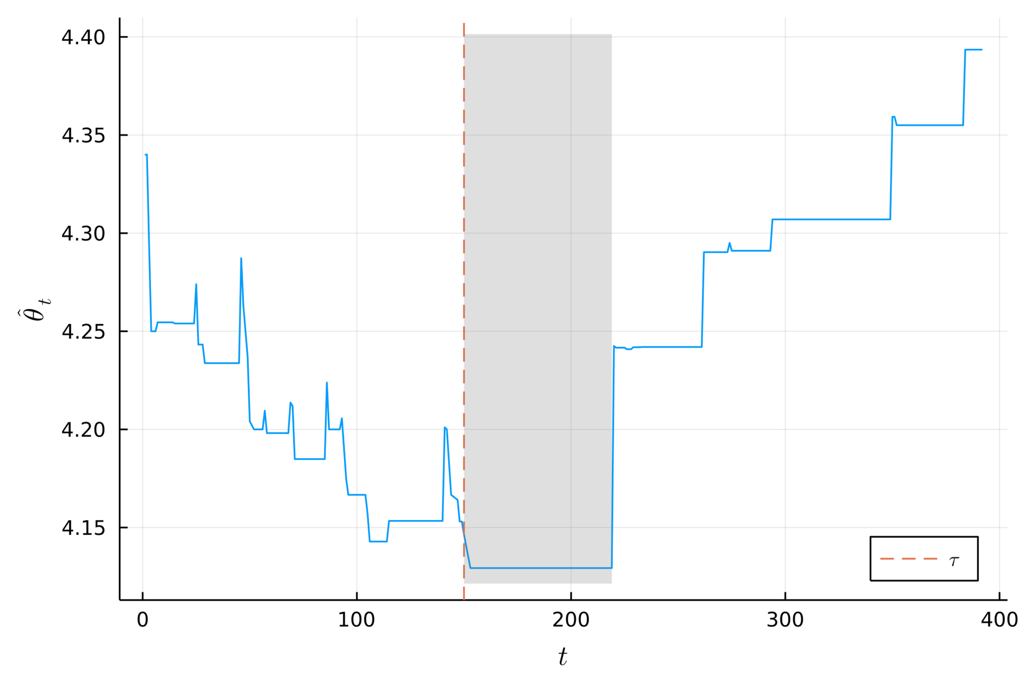

Figure 2. Resulting parameter estimate and "window of opportunity" (*shaded*).

Poisson data

- Detect **increases** in Poisson counts. For $\delta > 0$,

$$Y_t \sim \begin{cases} \mathrm{Pois}(\theta_0) & \text{for } t = 0, 1, \dots, \tau - 1 \\ \mathrm{Pois}(\theta_0 + \delta\sqrt{\theta_0})) & \text{for } t = \tau, \tau+1, \dots. \end{cases}$$

- Set $\theta_0 = 4$ and the design parameter $\lambda = 0.023$ to detect **small shifts** (optimal for $\delta = 0.25$).
- Estimate θ_0 with $m = 50$ observations and compute L such that

$$\mathbb{P}(\mathrm{ARL} \leq 500) = 0.1. \qquad (2)$$

- `CL` algorithm: **update $\widehat{\theta}$ only when $C_t = 0$.**

Stable-state performance

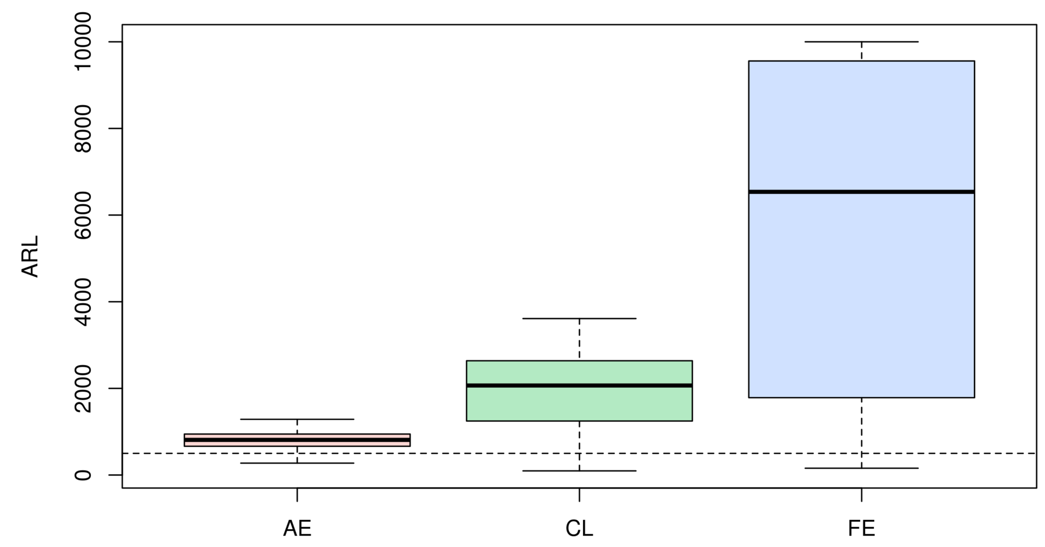

Figure 3. Conditional stable-state ARL of the EWMA control chart under fixed (FE), adaptive (AE), and cautious learning (CL) algorithms.

- `AE` has a lower critical value (Figure 4), whereas `FE` needs much larger critical values to satisfy (2).
- `CL`-type procedure: middle ground between the two algorithms.

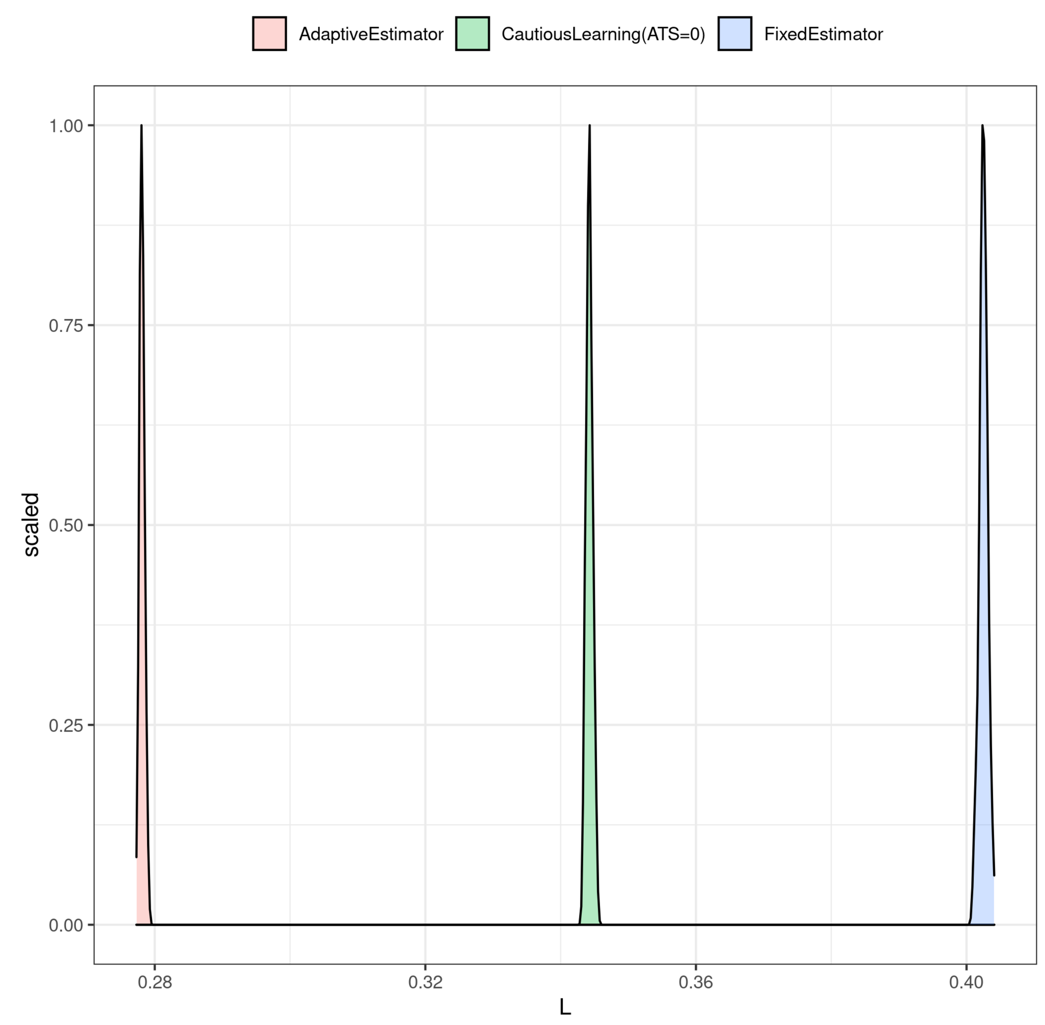

Figure 4. Distribution of the critical values using the fixed, adaptive, and cautious learning parameter update rules.

Remarks

There are two forces that work in opposite directions when considering control charts with critical limits obtained via the GICP design (1).

1. Bias of the estimate, which **lowers** the detection power of the scheme (Figure 2).
2. Precision of the estimate, which lowers the critical value and **increases** the detection power (Figure 3-4).

Performance under parameter shifts

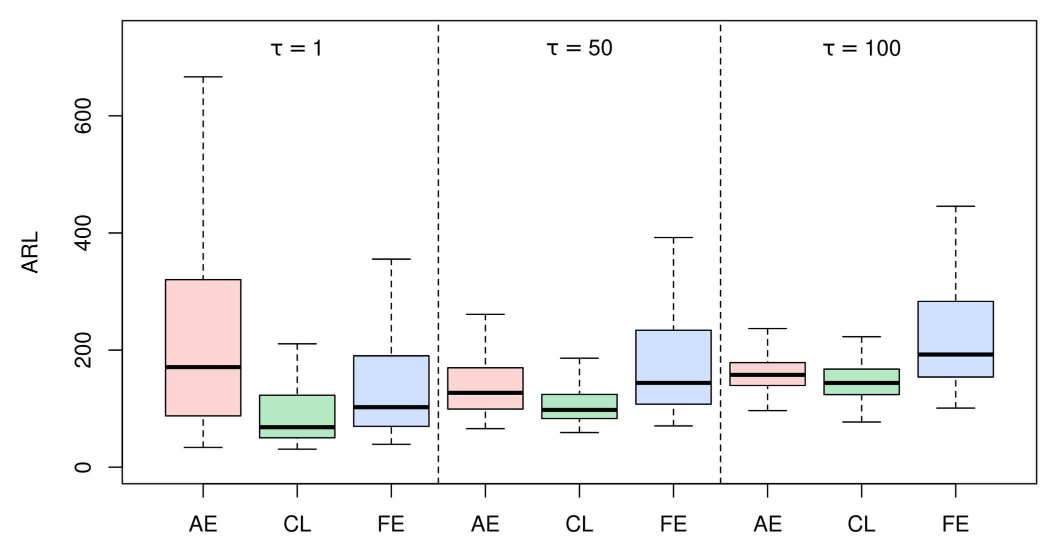

Figure 5. Conditional ARL of the EWMA control chart under fixed (FE), adaptive (AE), and cautious learning (CL) parameter updates against a *small shift* ($\delta = 0.35$).

- **Better overall performance** of the `CL` due to the balancing between precision and detection power.
- **Better detection** than the `FE` approach **even for very early shifts** ($\tau = 1$) because of the lower critical value (Figure 4).

Profile monitoring

- Detect **increases** in the model parameters for functional observations.

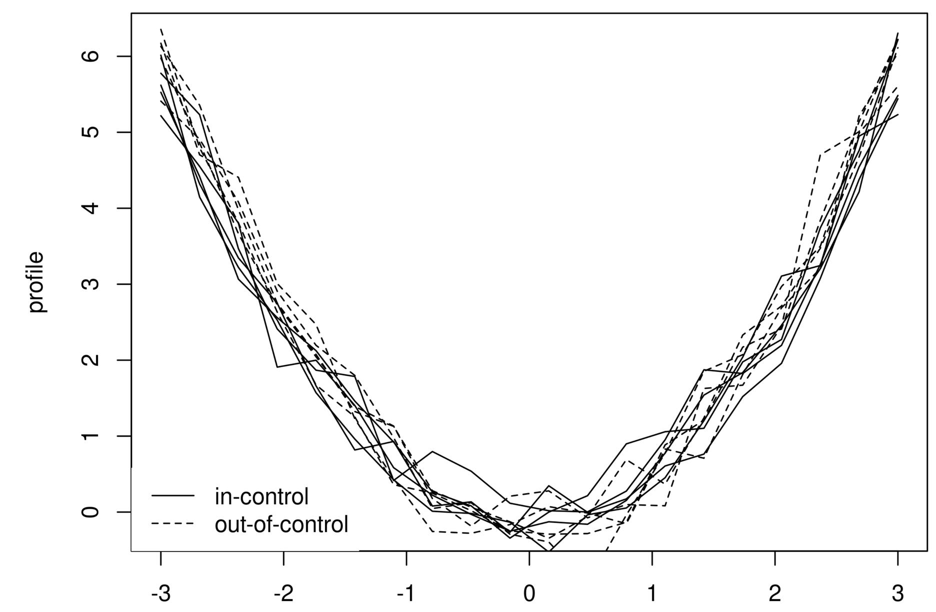

Figure 6. Simulated functional observations following a quadratic profile.

- Assume the following model for the i^{th} profile,

$$y_{t,j} = \alpha_0 + \beta_0 x_j^2 + \varepsilon_{t,j}, \quad j = 1, \dots, n.$$

- Apply the EWMA-type monitoring statistics to the reparametrization $\alpha^* = \alpha_0 + \beta_0\bar{x}^2$ and $\beta^* = s_{x^2}\beta_0$

$$A_t = \max\left\{0, (1-\lambda)A_{t-1} + \lambda(\widehat{\alpha}_t^* - \mathbb{E}_{\theta_0}[\widehat{\alpha}_t^*])/(\sqrt{\sigma^2/n})\right\},$$

$$B_t = \max\left\{0, (1-\lambda)B_{t-1} + \lambda(\widehat{\beta}_t^* - \mathbb{E}_{\theta_0}[\widehat{\beta}_t^*])/(\sqrt{\sigma^2/n})\right\}.$$

- Tuning parameter $\lambda = 0.033$ and call an alarm when either $A_t > L$ or $B_t > L$, where L is computed so that

$$\mathbb{P}(\mathrm{ARL} \leq 100) = 0.1.$$

- Set $\alpha_0 = 1$, $\beta_0 = 1$, and initial number of profiles $m = 20$ with $n = 10$ observations per profile.
- The `CL` rule updates each parameter estimates **when $A_t = 0$** and **when $B_t = 0$**, respectively.

Stable-state performance

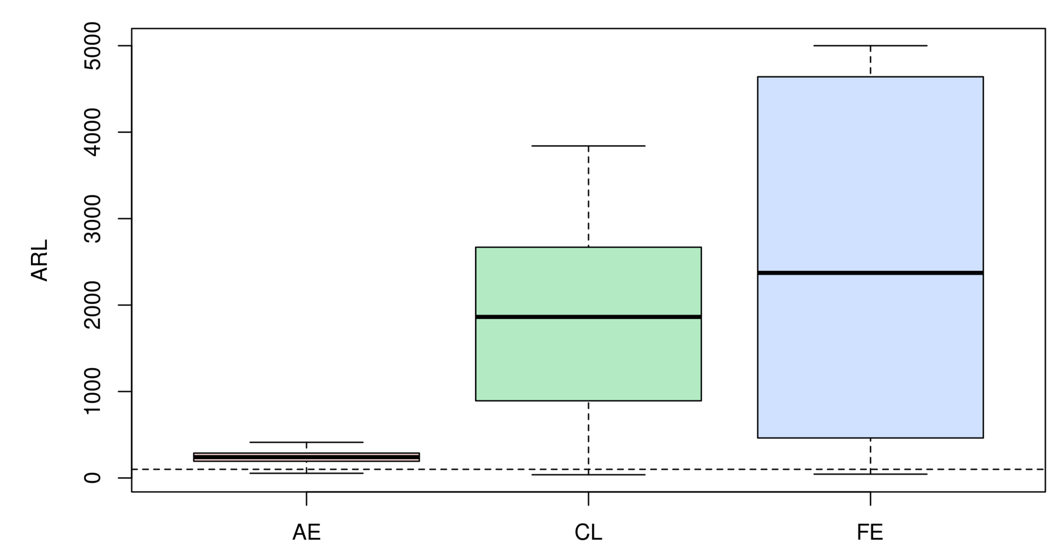

Figure 7. Conditional steady-state ARL of the EWMA control chart under fixed (FE), adaptive (AE), and cautious learning (CL) parameter updates.

Performance under parameter shifts

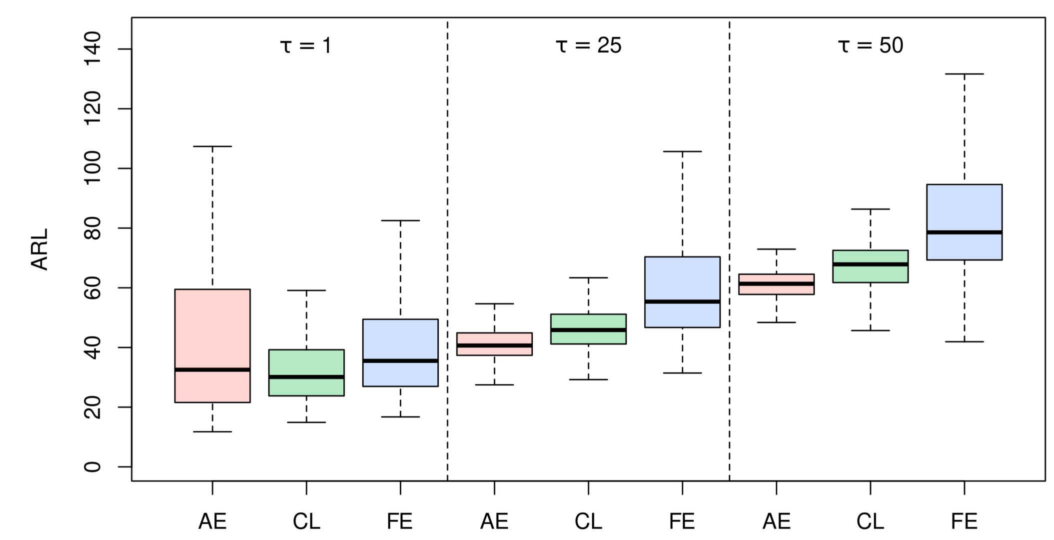

Figure 8. Conditional ARL of the EWMA control chart under fixed (FE), adaptive (AE), and cautious learning (CL) parameter updates against a *small shift* $\delta = 0.25$.

- The `CL` algorithm **is still the best choice** when $\tau = 1$, however the `AE` approach performs better for delayed shifts.

Conclusions

- A methodology which **generalizes the two most common learning rules** in statistical process control has been introduced.
- Simulations show that the proposed methodology can be used to **improve the detection performance, especially for early shifts**.

References

[1] G. Capizzi and G. Masarotto. "Guaranteed In-Control Control Chart Performance with Cautious Parameter Learning". In: *Journal of Quality Technology* 52.4 (2020).

[2] A. Gandy and J. T. Kvaløy. "Guaranteed Conditional Performance of Control Charts via Bootstrap Methods". In: *Scandinavian Journal of Statistics* 40.4 (2013).

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