Doubly-online changepoint detection for monitoring health status during sports activities

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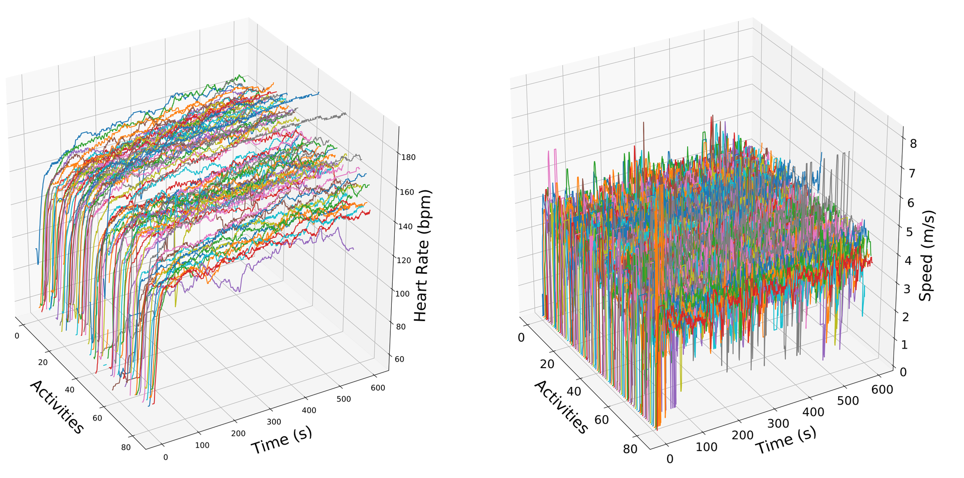
Today's monitoring of sports activities

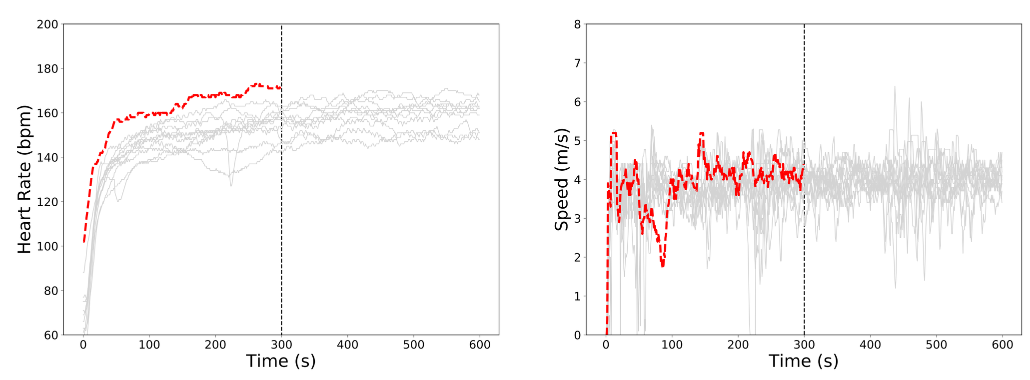
In sports monitoring, the use of smartwatches is widespread. Apps and wearable devices are indeed driving the next digital health and fitness revolution, in which intelligent and automatic real-time control and monitoring tools will become extremely relevant (Statista, 2020).

Every day people collect data about their sports activities as a sequence of multivariate time series with complex dependence structures, such as trends and periodic components.



However, people are not only interested in monitoring their sports activities day after day, but also while they are carried out.

We focus on real-time identification of variations in the behavior of one or more measurements caused, for example, by changes in physical condition.



The model

For one runner, we observe the data $\mathbf{y}_{1:N,1:T}$, composed of N ordered activities that are represented by P-dimensional times series at T time points.

We introduce $S_{1:N} = (S_1, \dots, S_N)$ to divide the N activities in S_N segments, according to a discrete Markov Chain with transition probability

$$p(S_n|S_{n-1}) = \begin{cases} \lambda, & \text{if } S_n = S_{n-1}+1, \\ 1-\lambda, & \text{if } S_n = S_{n-1}, \end{cases}$$

for $\lambda \in (0,1)$ and $S_1 = 1$.

Assume that activity n belongs to segment s. We model its measurements at time t with a state space representation with measurement equation

$$\mathbf{y}_{n,t} = \begin{bmatrix}\mathbf{Z}_{\boldsymbol{\theta}}^{(S)} & \mathbf{Z}_{\boldsymbol{\theta}}^{(A)}\end{bmatrix} \begin{bmatrix}\boldsymbol{\alpha}_t^{(s)} \\ \boldsymbol{\alpha}_{n,t}\end{bmatrix} + \boldsymbol{\epsilon}_{n,t},$$

with $\boldsymbol{\epsilon}_{n,t} \overset{iid}{\sim} N_P(\mathbf{0}, \boldsymbol{\Sigma}_{\boldsymbol{\theta}})$, and state equation

$$\begin{bmatrix}\boldsymbol{\alpha}_{t+1}^{(s)} \\ \boldsymbol{\alpha}_{n,t+1}\end{bmatrix} = \begin{bmatrix}\mathbf{T}_{\boldsymbol{\theta}}^{(S)} & \mathbf{0} \\ \mathbf{0} & \mathbf{T}_{\boldsymbol{\theta}}^{(A)}\end{bmatrix} \begin{bmatrix}\boldsymbol{\alpha}_t^{(s)} \\ \boldsymbol{\alpha}_{n,t}\end{bmatrix} + \begin{bmatrix}\boldsymbol{\eta}_t^{(s)} \\ \boldsymbol{\eta}_{n,t}\end{bmatrix},$$

with $\boldsymbol{\eta}_t^{(s)} \overset{iid}{\sim} N_M(\mathbf{0}, \boldsymbol{\Psi}_{\boldsymbol{\theta}})$, $\boldsymbol{\eta}_{n,t} \overset{iid}{\sim} N_K(\mathbf{0}, \boldsymbol{\Delta}_{\boldsymbol{\theta}})$, and $\boldsymbol{\alpha}_1^{(s)} \overset{iid}{\sim} N_M(\hat{\boldsymbol{\alpha}}_{1|0}^{(S)}, \mathbf{P}_{1|0}^{(S)})$ independent of $\boldsymbol{\alpha}_{n,1} \overset{iid}{\sim} N_K(\hat{\boldsymbol{\alpha}}_{1|0}^{(A)}, \mathbf{P}_{1|0}^{(A)})$.

The likelihood

We connect our model to the work by Yildirim, Singh and Doucet (2013), by considering the *delays* from the last changepoint, i.e.

$$D_n|D_{n-1} = \begin{cases} D_{n-1}+1, & \text{if } S_n = S_{n-1}, \\ 1, & \text{if } S_n = S_{n-1}+1, \end{cases}$$

with $D_1 = 1$, to express the likelihood as

$$p_{\boldsymbol{\theta}}(\mathbf{y}_{1:N,1:T}) = \mathsf{E}_{\boldsymbol{\theta}}\Big[\prod_{n=1}^{N} G_{\boldsymbol{\theta},n}^{D}(D_n)\Big],$$

where the potential $G_{\boldsymbol{\theta},n}^{D}(D_n) = p_{\boldsymbol{\theta}}(\mathbf{y}_{n,1:T}|D_{1:n}, \mathbf{y}_{1:(n-1),1:T})$ is defined as

$$G_{\boldsymbol{\theta},n}^{D}(D_n) = \begin{cases} \frac{p_{\boldsymbol{\theta}}(\mathbf{y}_{j:n,1:T}|D_n)}{p_{\boldsymbol{\theta}}(\mathbf{y}_{j:(n-1),1:T}|D_{n-1})}, & \text{if } D_n = D_{n-1}+1, \\ p_{\boldsymbol{\theta}}(\mathbf{y}_{n,1:T}|D_n), & \text{if } D_n = 1, \end{cases}$$

with $j = n - D_n + 1$.

Between-online changepoint detection via EM

The between-online setting aims at maximizing the likelihood to detect changepoint whenever a new activity is fully observed.

We adopt the online EM algorithm for changepoint detection proposed by Yildirim, Singh and Doucet (2013), which is based on the *forward smoothing technique* (see, e.g., Kantas et al., 2015).

SMC approximation of the predicted probabilities

The forward smoothing technique requires to compute $p_{\boldsymbol{\theta}}(D_{n-1}|D_n, \mathbf{y}_{1:(n-1),1:T})$ and $p_{\boldsymbol{\theta}}(D_n|\mathbf{y}_{1:n,1:T})$. Both these quantities depend on the predicted probability $p_{\boldsymbol{\theta}}(D_n|\mathbf{y}_{1:(n-1),1:T})$, obtained with SMC.

Let $\eta_{n-1}^B(D_{n-1})$ be a particle approximation of $p_{\boldsymbol{\theta}}(D_{n-1}|\mathbf{y}_{1:(n-2),1:T})$, composed of B particles with support $\mathcal{D}_{n-1}^B = \{d_{n-1}^1, \dots, d_{n-1}^{B-1}, d_{n-1}^B\}$ composed by the particles themselves.

We consider the *augmented* support $\mathcal{D}_n^{B\star}$, of size $2B$, with generic elements $(1, d_{n-1}^b)$ and (d_{n-1}^b+1, d_{n-1}^b), and sample B particles from $\mathcal{D}_n^{B\star}$ with weights

$$W(D_n, D_{n-1}) \propto p(D_n|D_{n-1}) G_{\boldsymbol{\theta},n-1}^{D}(D_{n-1}) \eta_{n-1}^B(D_{n-1})$$

to obtain $\eta_n^B(D_n) = \sum_{b=1}^B \delta_{D_n}(d_n^b, d_{n-1}^b)$, with support $\mathcal{D}_n^B = \{d_n^1 \dots, d_n^B\}$, where $\delta_{D_n}(d_n^b, d_{n-1}^b) = 1$ if $D_n = d_n^b$, and 0 otherwise.

The within-online setting

The ability to monitor the presence of a changepoint during activity n is given by the need of computing, on the fly, the filtered probability

$$p_{\boldsymbol{\theta}}(D_n|\mathbf{y}_{n,1:t}, \mathbf{y}_{1:(n-1),1:T}),$$

for any $t < T$.

It is easy to show that this filtered probability is proportional to

$$p_{\boldsymbol{\theta}}(\mathbf{y}_{n,1:t}|D_n, \mathbf{y}_{1:(n-1),1:T}) p_{\boldsymbol{\theta}}(D_n|\mathbf{y}_{1:(n-1),1:T}),$$

where $p_{\boldsymbol{\theta}}(D_n|\mathbf{y}_{1:(n-1),1:T})$ is approximated by $\eta_n^B(D_n)$ and

$$p_{\boldsymbol{\theta}}(\mathbf{y}_{n,1:t}|D_n, \mathbf{y}_{1:(n-1),1:T}) = \begin{cases} \frac{p_{\boldsymbol{\theta}}(\mathbf{y}_{n,1:t}, \mathbf{y}_{j:(n-1),1:T}|D_n)}{p_{\boldsymbol{\theta}}(\mathbf{y}_{j:(n-1),1:T}|D_n)}, & \text{if } D_n > 1, \\ p_{\boldsymbol{\theta}}(\mathbf{y}_{n,1:t}|D_n), & \text{if } D_n = 1, \end{cases}$$

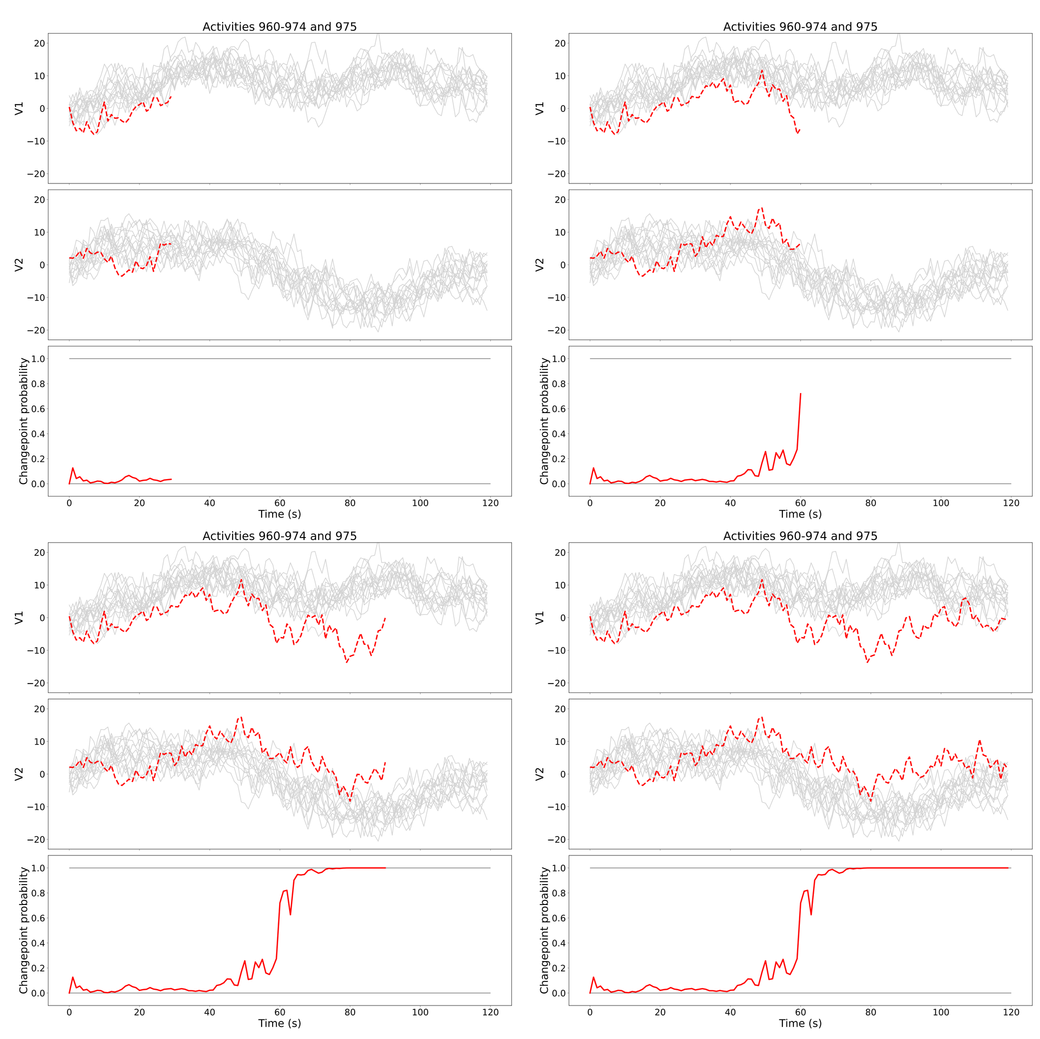
with $j = \max(1, n - D_n + 1)$.

Recognizing the elements that can be computed before the activity begins is essential to effectively monitor it in real-time environments.

While $p_{\boldsymbol{\theta}}(\mathbf{y}_{j:(n-1),1:T}|D_n)$ can be computed before the activity starts, $p_{\boldsymbol{\theta}}(\mathbf{y}_{n,1:t}, \mathbf{y}_{j:(n-1),1:T}|D_n)$ needs to be evaluated during the activity.

Real-time monitoring of changepoint probabilities

Monitoring the probability of changepoint while one activity is performed allows to quantify the uncertainty related to changes in the health status of the athlete.

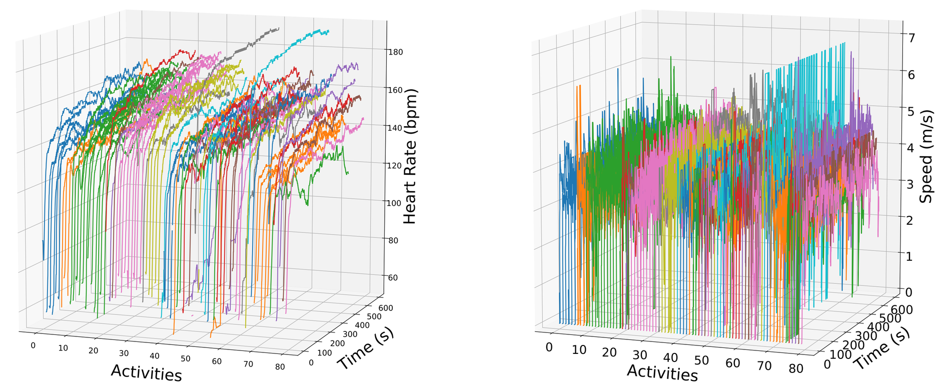


Probability of changepoint at time $t = 30, 60, 90, 120$.

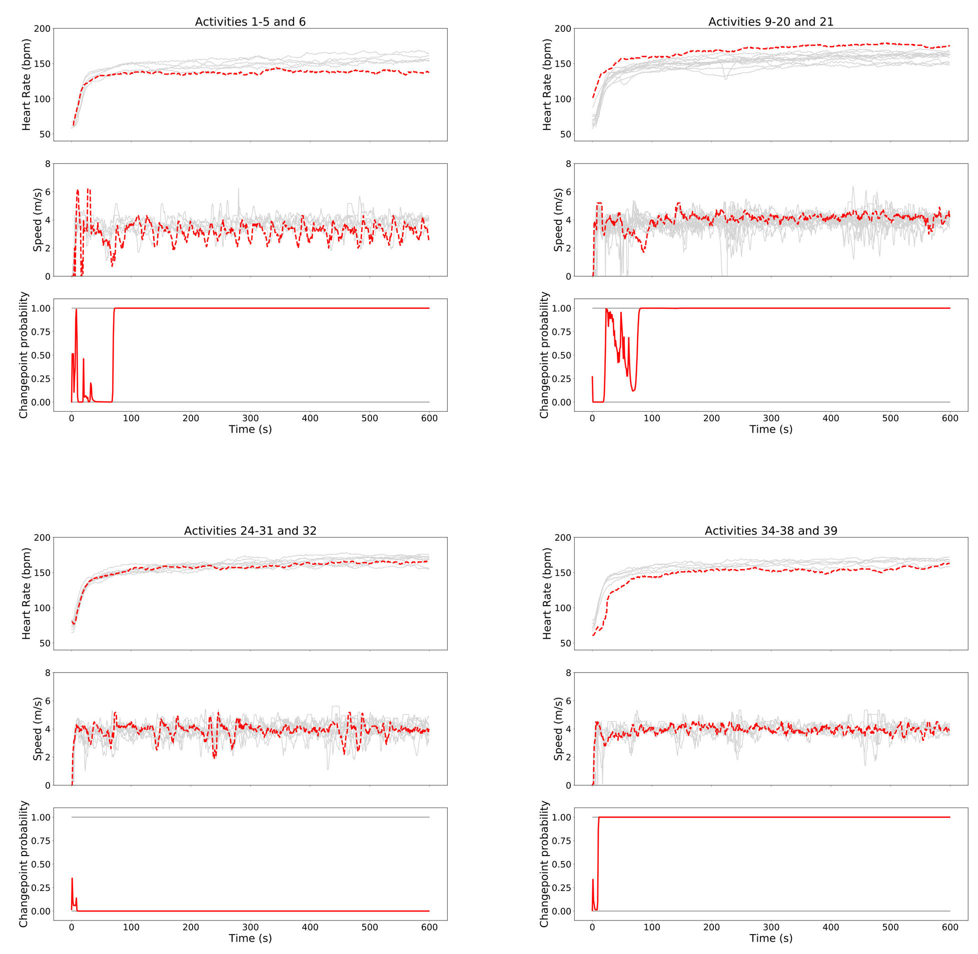
Real data application

We consider a set of 85 warm-up running activities on flat routes consisting of the first 10 minutes of running of a well-trained athlete, and monitor the heart rate (internal load, bpm) and the speed (external load, m/s) during the activities.

The heart rate provides insights on the oxygen consumption during the activity, but can be influenced by the intensity of the exercise, represented by the speed of the runner.



With a probability threshold of 0.5 the number of detected changepoints is 39. This highlights the high variability of the activities.



With our model, we can derive information on the activities while they are carried out. In the figures, four different activities are classified as changepoint and not a changepoint according to their behaviors with respect to the past.

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About this work

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You might be interested in my works on the topic:

Stival, M., Bernardi, M., Dellaportas, P. (2022). Doubly-online changepoint detection for monitoring health status during sports activities. *arXiv preprint arXiv:2206.11578*.

Stival, M., Bernardi, M., Cattelan, M., Dellaportas, P. (2022). Missing data patterns in runners' careers: do they matter? *arXiv preprint arXiv:2206.12716* .

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